Time To Intervene: A Continuous-Time Approach to Network Analysis and Centrality

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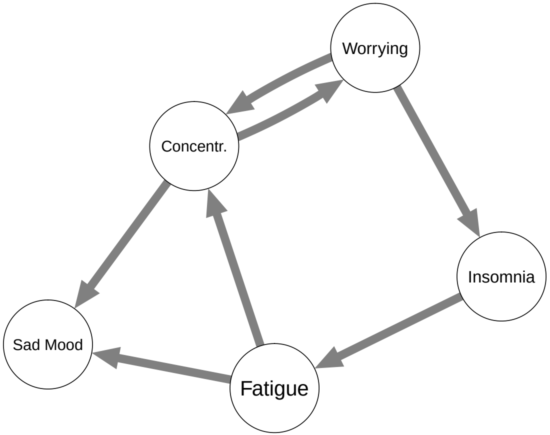
Utrecht University

EAM 2021

Network Approach to Psychopathology



(Cramer at al, 2010; Borsboom et al. 2013; Schmittman et al, 2013; Borsboom, 2017)

Dynamical Network Analysis

$$\boldsymbol{Y}_{\tau+1} = \boldsymbol{\Phi}\boldsymbol{Y}_{\tau} + \boldsymbol{\epsilon}_{\tau}$$

(Bringmann et al. 2013; Fisher et al. 2017)

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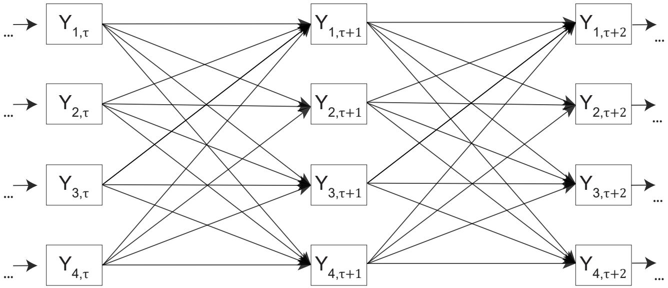

(Bringmann et al. 2013; Fisher et al. 2017; figure from Ryan & Hamaker, 2021)

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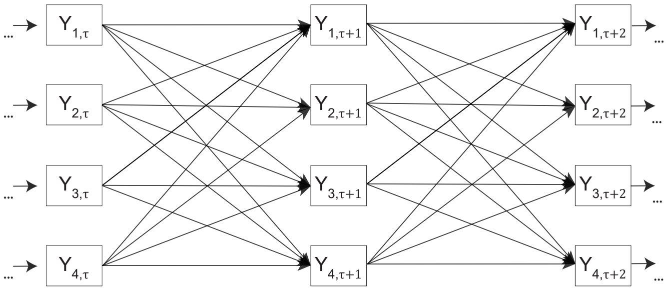

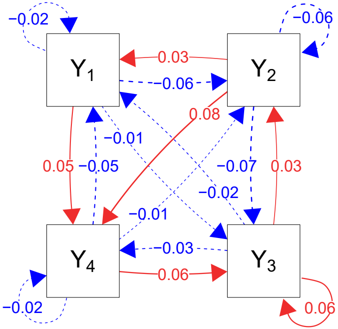

(Bringmann et al. 2013; Fisher et al. 2017; figure from Ryan & Hamaker, 2021)

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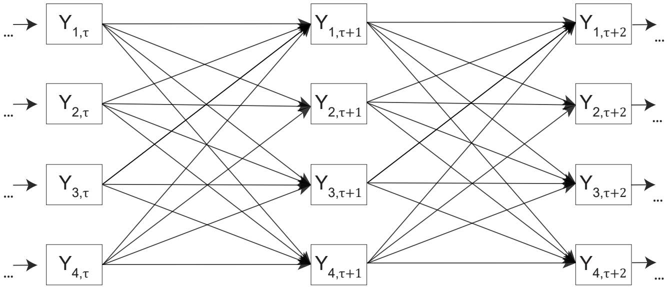

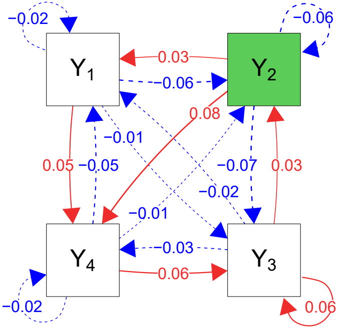

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The Time-Interval Problem

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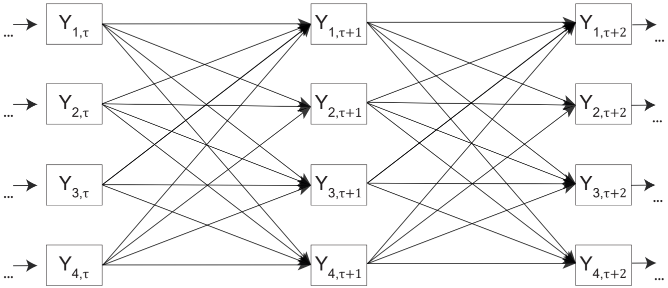

(Gollob & Reichardt, 1987; Singer 2012; Voelkle, Oud, Davidov, Schmidt 2013; Kuiper & Ryan, 2018)

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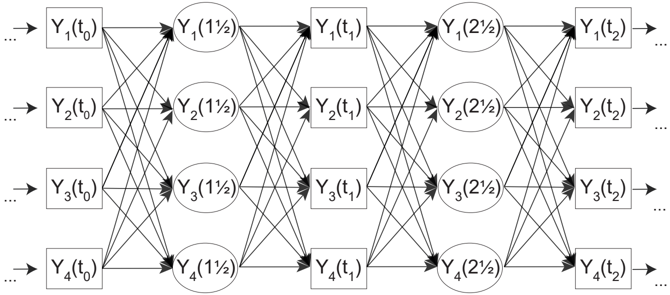

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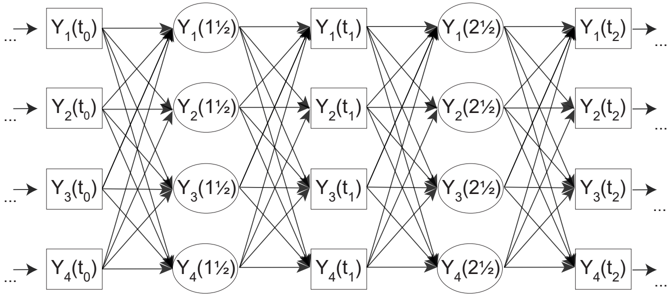

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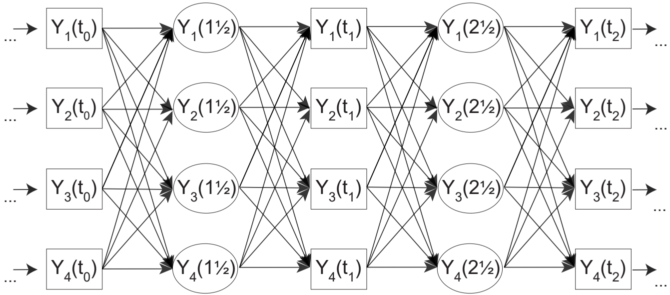

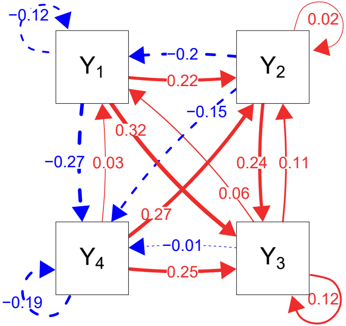

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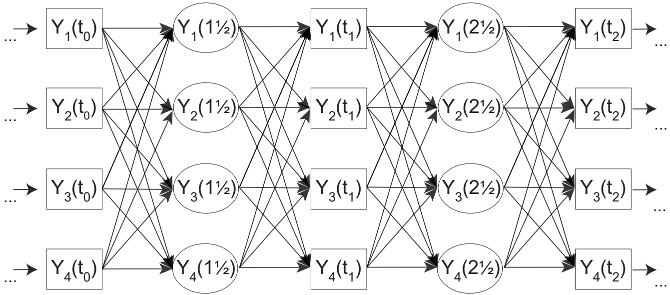

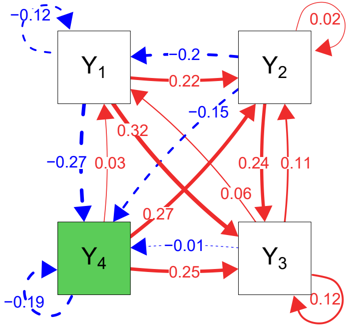

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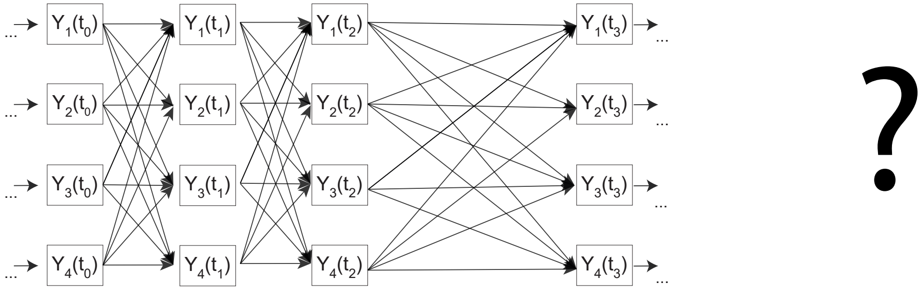

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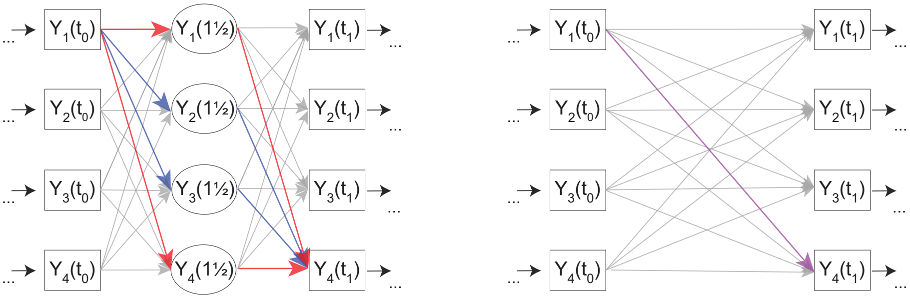
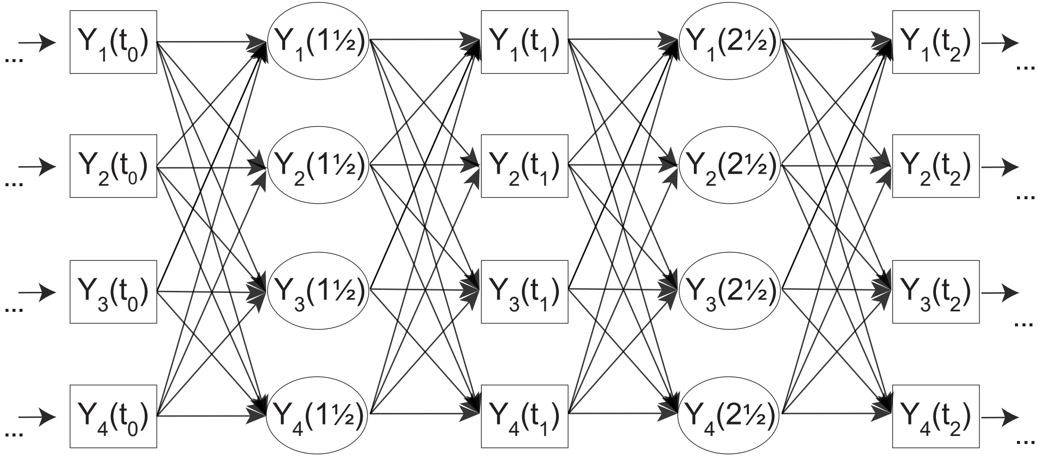


(Singer 2012; Deboeck & Preacher 2013; Ryan & Hamaker, 2021)



$Y_1(t_0)$
$Y_2(t_0)$
$Y_3(t_0)$
$Y_4(t_0)$
$Y_1(1½)$
$Y_2(1½)$
$Y_3(1½)$
$Y_4(1½)$
$Y_1(t_1)$
$Y_2(t_1)$
$Y_3(t_1)$
$Y_4(t_1)$
$Y_1(2½)$
$Y_2(2½)$
$Y_3(2½)$
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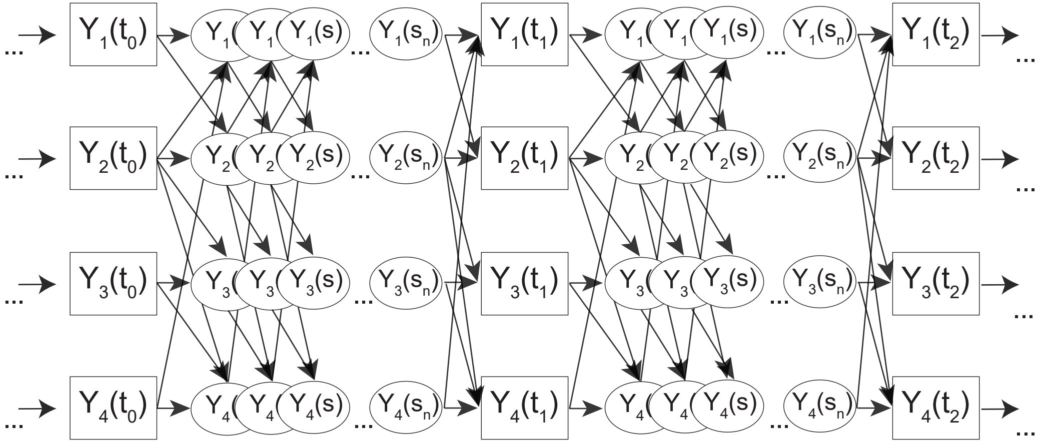
$Y_1(t_2)$
$Y_2(t_2)$
$Y_3(t_2)$
$Y_4(t_2)$



$Y_1(t_0)$
$Y_2(t_0)$
$Y_3(t_0)$
$Y_4(t_0)$
$Y_1(s)$
$Y_2(s)$
$Y_3(s)$
$Y_4(s)$
$Y_1(s_n)$
$Y_2(s_n)$
$Y_3(s_n)$
$Y_4(s_n)$
$Y_1(t_1)$
$Y_2(t_1)$
$Y_3(t_1)$
$Y_4(t_1)$
$Y_1(t_2)$
$Y_2(t_2)$
$Y_3(t_2)$
$Y_4(t_2)$

Continuous-Time Models

$$\frac{d\boldsymbol{Y}(t)}{dt} = \boldsymbol{A}\ \boldsymbol{Y}(t) + \boldsymbol{G}\frac{d\boldsymbol{W}(t)}{dt}$$

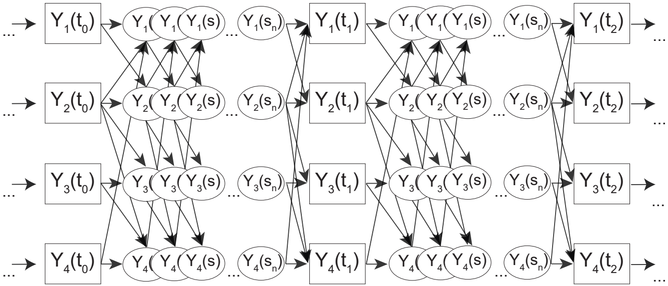


Figure from Ryan & Hamaker (2021)

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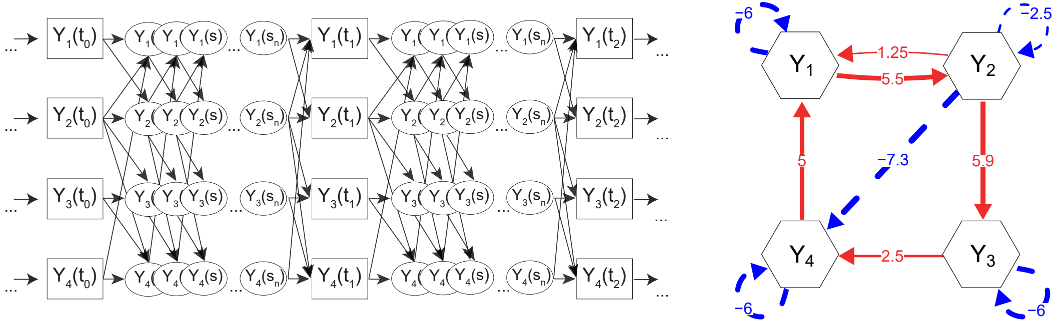
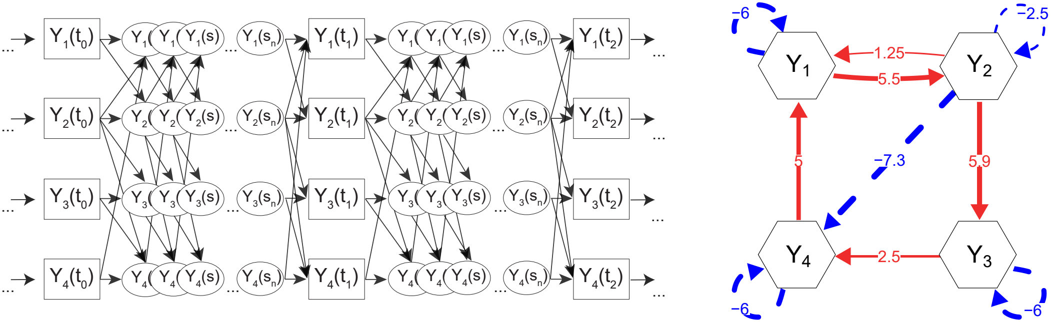


Figure from Ryan & Hamaker (2021)

Continuous-Time Models

$$\boldsymbol{Y}(t + \Delta t) = e^{\boldsymbol{A}\Delta t}\boldsymbol{Y}(t) + \epsilon(\Delta t)$$



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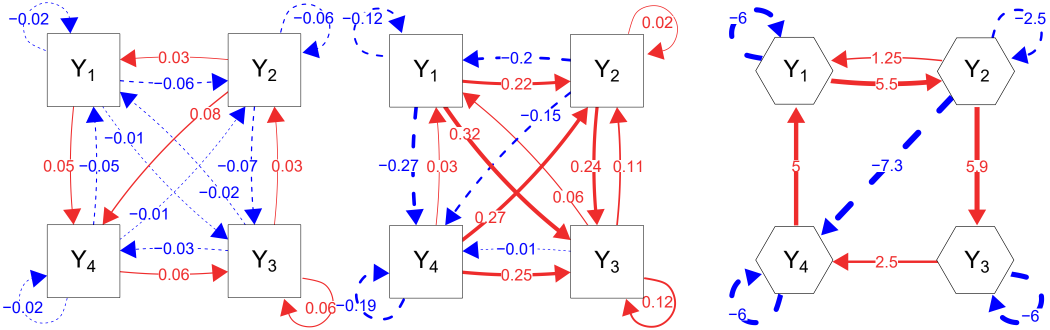

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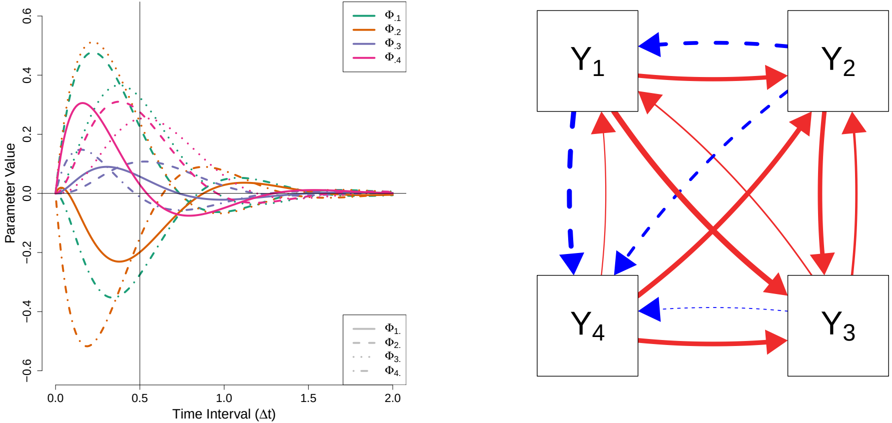

Parameter Value
Time Interval (Δt)
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
0.0
0.5
1.0
1.5
2.0
$\Phi_{.1}$
$\Phi_{.2}$
$\Phi_{.3}$
$\Phi_{.4}$
$\Phi_{1.}$
$\Phi_{2.}$
$\Phi_{3.}$
$\Phi_{4.}$
Y_1
Y_2
Y_3
Y_4

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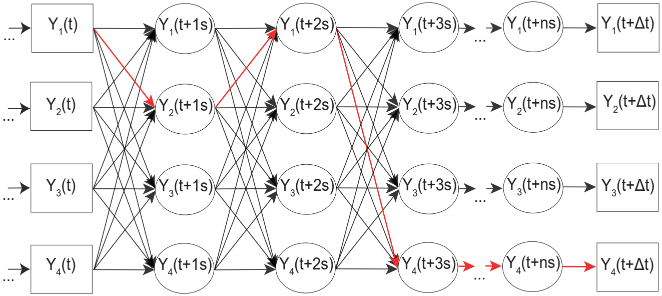

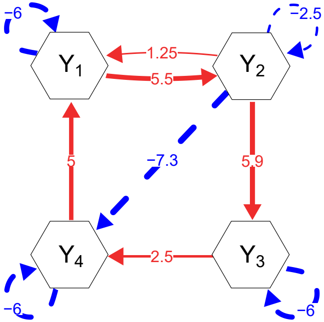

(Deboeck & Preacher, 2013; Ryan & Hamaker 2021)

Intervention Targets?

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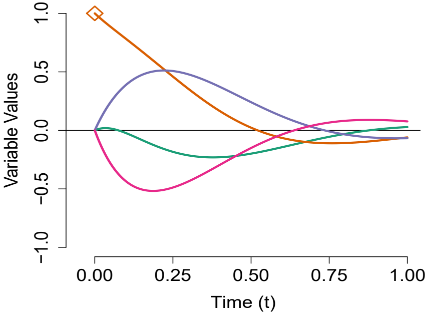
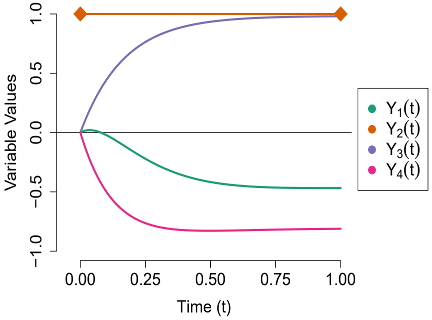
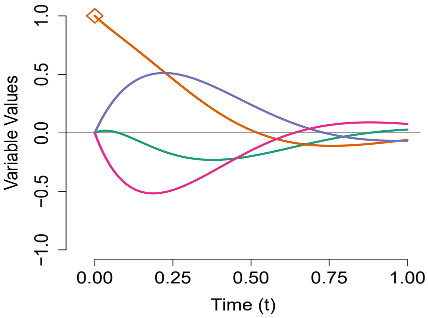


Pulse Intervention



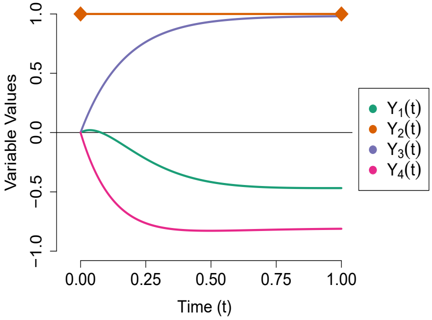
Press Intervention

Intervention Targets?



Pulse Intervention

Total Effect Centrality

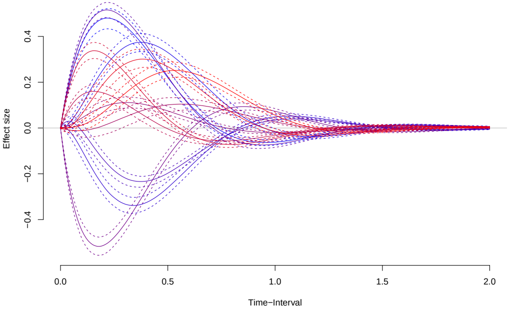


Press Intervention

Indirect Effect Centrality

A Continuous-Time Approach to Network Analysis

- The Time-Interval Problem
- Continuous-Time Models
  - *ctsem*, *dynr*
- CT network analysis and centrality
  - *ctnet* (github: ryanoisin/ctnet)
- Conditions for Causal Inference?
- Theoretical Models? (Haslbeck*, Ryan*, Robinaugh* et al. 2021)



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Exponential as Path-Tracing

$$\frac{dY(t)}{dt} = A \;\; Y(t)$$

$$\lim_{s \to 0} \frac{Y(t+s) - Y(t)}{s} = A \;\; Y(t)$$

$$\lim_{s \to 0} Y(t+s) = (1 + A \lim_{s \to 0} s) Y(t)$$

$$Y(t + \Delta t) = \lim_{n \to \infty} (I + A \frac{\Delta t}{n})^n Y(t)$$

$$Y(t + \Delta t) = e^{A \Delta t} Y(t)$$