Lord's paradox

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Causality

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One way in which researchers have tried to study this is through a **pre-post test design**, in which the (potential) cause x is measured once, and the outcome is measured twice (y_1 and y_2).

Two broad classes of models

1: Change score method

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also known as: *pretest-postest covariance* or *covariance adjusted score* (when x_1 is dichotomous), or as *cross-lagged panel analysis*.

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An **alternative expression** of the second model is:

2: Baseline-adjusted gain scores

$$y_2 - y_1 = \gamma_0 + \gamma_1 x_1 + (\gamma_2 - 1) y_1 + \upsilon$$

also known as: *residualized gain scores* or *residual change*.

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	Antisocial behavior	
Professional interventions		
Psychotherapy visits	.07**	.00
Ritalin	.07**	.04
Parental disciplinary actions		
Non-physical punishment	.03	−.08**
Physical punishment	.07**	−.05
Scolding/yelling	.06*	−.08**
“Hostile/ineffective” scale	.09**	−.15**

This shows that:

- regressor variable method (first column): **adverse** effect (or no effect)
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So what is the truth?

Bad reputation of change scores

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1. **Unreliability**: $y_2 - y_1$ tends to be (much) less reliable than y_1 and y_2

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1. **Unreliability**: $y_2 - y_1$ tends to be (much) less reliable than y_1 and y_2

2. **Regression towards the mean**:

   - $y_2 - y_1$ is typically negatively correlated with y_1 (people high on y_1 will decrease and those low on y_1 will increase)
   - if x_1 is correlated with y_1, it will have a spurious relationship with $y_2 - y_1$

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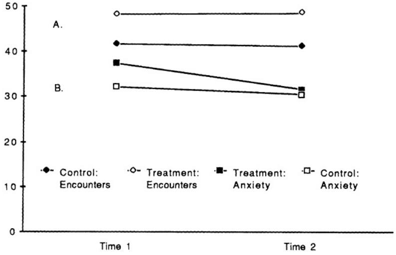

FIGURE 1 Means from Table 1.

Example A (encounters): Regressor variable method **erroneously detects a difference in change** between the groups (suggesting treatment had a detrimental effect)

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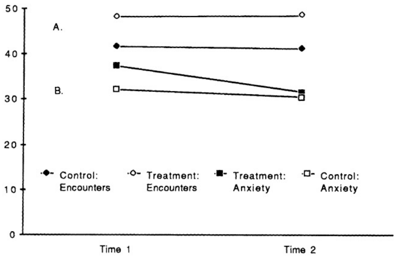

FIGURE 1 Means from Table 1.

Example A (encounters): Regressor variable method **erroneously detects a difference in change** between the groups (suggesting treatment had a detrimental effect)

Example B (anxiety): Regressor variable method **fails to detect a difference in change** between the groups (suggesting that treatment does not decrease anxiety)

Repeated measures models

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Advice for four scenarios by van Breukelen:

- random assignment: use Regressor variable method (more power)
- assignment (entirely!) dependent on pretest score: use Regressor variable method (Change score is biased)
- assignment based on preexisting/natural groups: do not use Regressor variable method; Change score method might be right (requires the assumption that both groups change by the same amount when there is no treatment)
- self-assignment: unclear

Advice for pre-existing groups

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Note that this all (seems to) generalize to the case where x is a **continuous variable**, measured **simultaneously** with y_1.

Model = truth?

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There is a large body of literature based on the idea that there is **unobserved heterogeneity** (i.e., stable between-person, trait-like differences), like:

$$y_{it} = \beta_0 + \alpha_i + \beta_1 x_{it} + \varepsilon_{it}$$

where α_i captures **unobserved omitted variables that are invariant over time**.

Question: What is **the effect of the diet** provided by university dinning halls on students’ weight, and **are there sex differences in these effects**?

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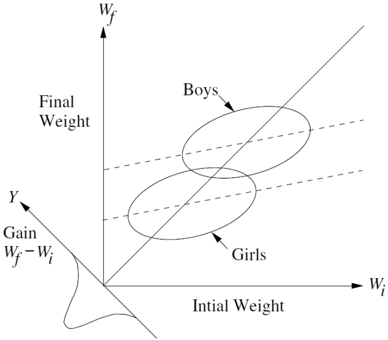
Basics: This is a **pre-post test design** with **two existing groups** (boys and girls).

Hence, the **"treatment" is not the diet** (as this is the same for everyone), **but gender**: Do gender differences in metabolism have a different effect on the weight of boys than on the weight of girls?



W_f
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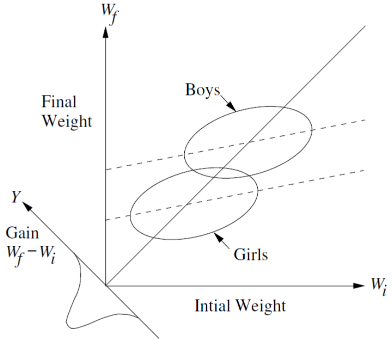
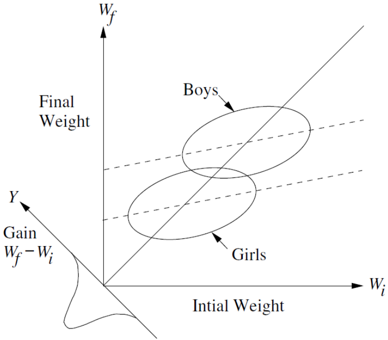


- Mean of girls has not changed; mean of boys has not changed



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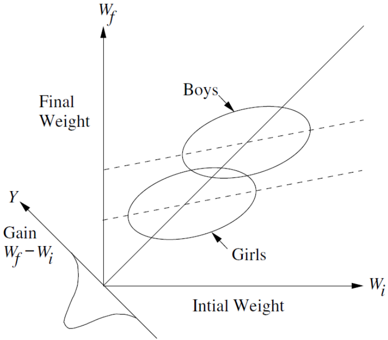
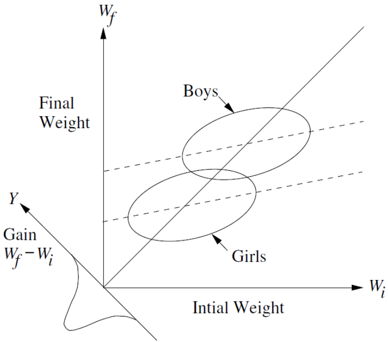


- Mean of girls has not changed; mean of boys has not changed
- Frequency distributions within groups has not changed
- **Conclusion**: while there are individual changes, overall there are no changes for either boys or girls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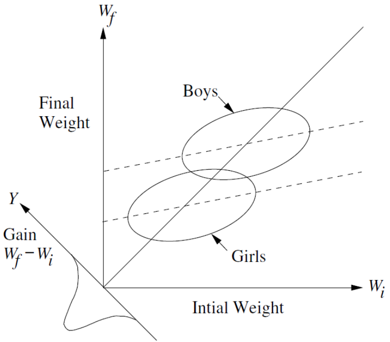
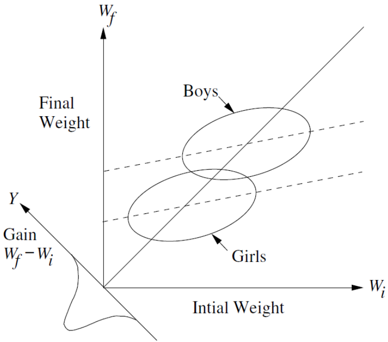
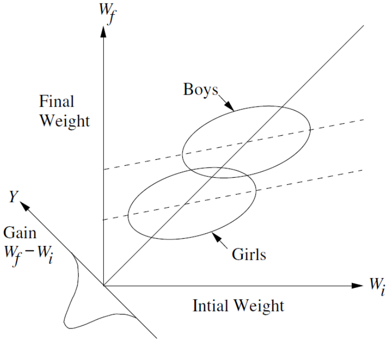


- ANCOVA with initial weight as covariate and gender as the factor



- ANCOVA with initial weight as covariate and gender as the factor
- **Conclusion**: the weight gain for boys is larger than that for girls, when proper allowance for initial weight is made (see the difference in intercepts)

Paradox



When the question is: **Is there differential gain**?

- there are no changes in mean for either group; hence **NO differential gain**
- when boys and girls start with the same weight, the boys will gain more than the girls; so **there is differential gain**

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- weight gain ($\Delta W_j = W_{f,j} - W_{i,j}$) is the outcome
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Statistician 1 looks at the **total effect of gender** (with dummy variable M_j for males) on weight gain:

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Statistician 2 looks at the **direct effect of gender** on weight gain:

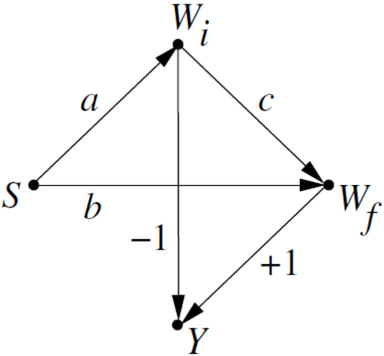
$$\Delta W_j = b_0 + b_1 M_j + b_2 W_{i,j} + e_j$$

which can be expressed as the ANCOVA model:

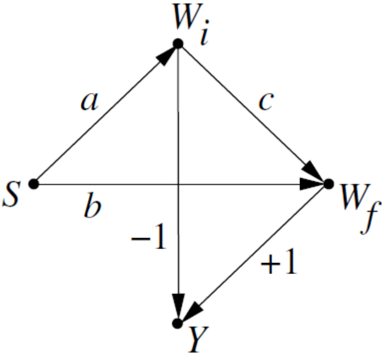
$$W_{f,j} = b_0 + b_1 M_j + (b_2 + 1) W_{i,j} + e_j$$

And now with a DAG

- Cause is sex (S)
- Outcome is weight gain ($Y = W_f - W_i$)
- Mediator is initial weight (W_i)



The two answers based on the DAG



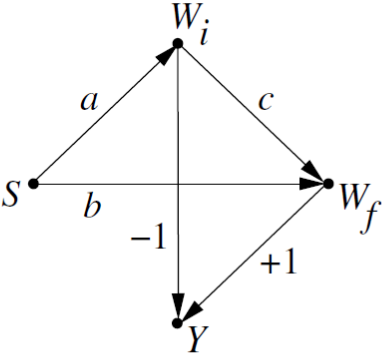
Total effect: multiply all coefficients of a path from S to Y, and sum these

$$TE = b*1 + a*c*1 + a*(-1) = b - a(1-c)$$

Direct effect: consider only paths that do not contain the mediator

$$DE = b*1$$

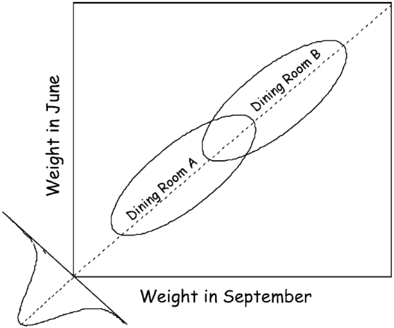
In words

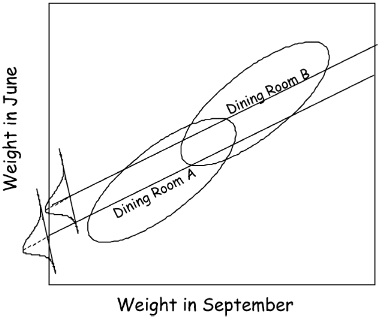


No total effect: $b - a(1-c) = 0$
Positive direct effect: $b > 0$
On average a boy gains more than a girl **of equal initial weight** ($b > 0$), but since there are more heavy-weight boys than girls and we subtract a portion of this difference, overall the gain for boys is the same as the gain for girls.

Conclusion: There is no paradox!

Different diets (instead of sex)



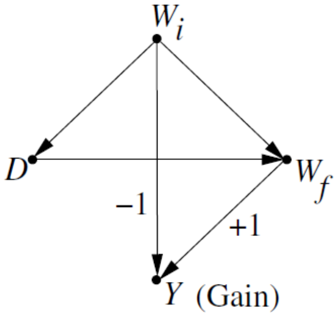


Group means are again on the 45-degree line: no mean changes over time in either group.

ANCOVA results in different intercepts for the two groups: More weight gain in Dining Room B.

Critical here is that **heavier students tended to choose dining room B** more often.

Pearl: Now it is confounding, not mediation



- Initial weight is **no longer the mediator**; it is now the first variable in the causal sequence.
- It is a common cause or **confounder** of the relationships between (potential) cause (dinning room) and outcome (final weight or weight gain).
- We **need to control for this**; failing to do so biases the results

The role of the pre-test score in the DAG

So the critical distinction is: Is the pre-test score a mediator (affected by the potential cause of interest), or a confounder (affecting the potential cause of interest)?

Draw the **DAGs for these scenarios**:

- Larzelere: Pre- and post-test measures of deviant behavior; potential cause is parental discipline
- Allison: Pre- and post-test of number of social encounters; groups are children with facial abnormalities and controls; first group is treated between pre-test and post-test