

Mapping Statistical Models to Causal Structures

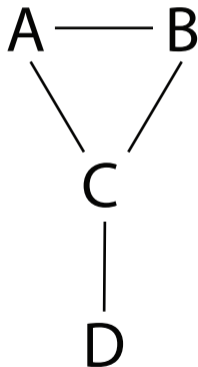
Oisín Ryan

Department of Methodology and Statistics
Utrecht University,
The Netherlands

May 6, 2019

Project 1: Undirected Networks and Causal Skeletons ¹

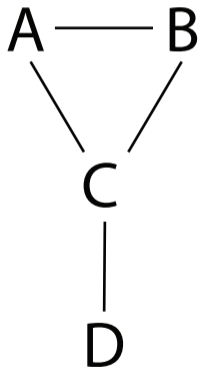
Undirected Network



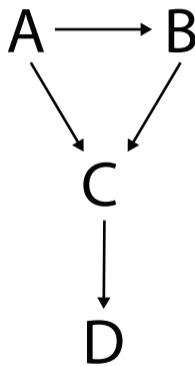
¹Borsboom & Cramer (2013), van Borkulo et al. (2015), Boschloo et al. (2016), Fried et al. (2016)

Project 1: Undirected Networks and Causal Skeletons ¹

Undirected Network



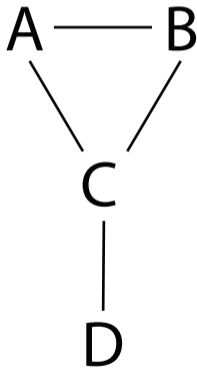
Directed Causal Structure



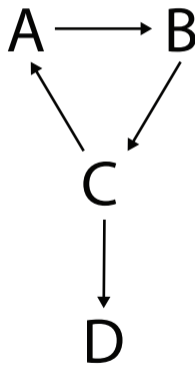
¹Borsboom & Cramer (2013), van Borkulo et al. (2015), Boschloo et al. (2016), Fried et al. (2016)

Project 1: Undirected Networks and Causal Skeletons

Undirected Network

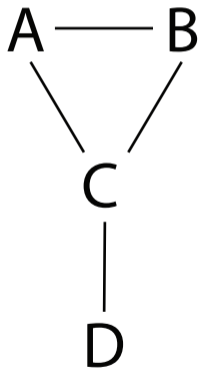


Directed Causal Structure

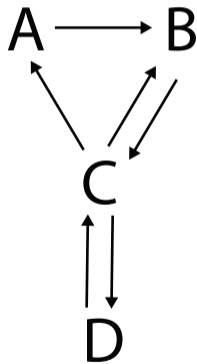


Project 1: Undirected Networks and Causal Skeletons

Undirected Network

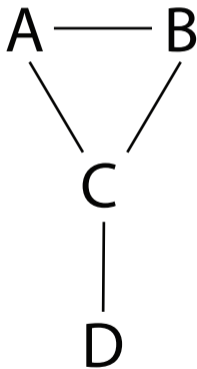


Directed Causal Structure

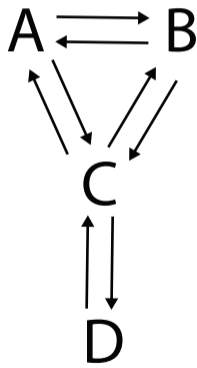


Project 1: Undirected Networks and Causal Skeletons

Undirected Network

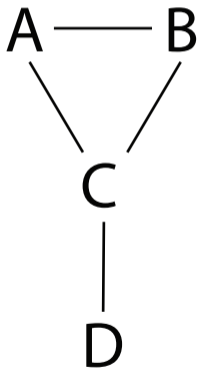


Directed Causal Structure

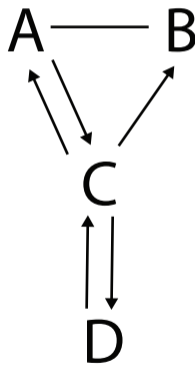


Project 1: Undirected Networks and Causal Skeletons

Undirected Network

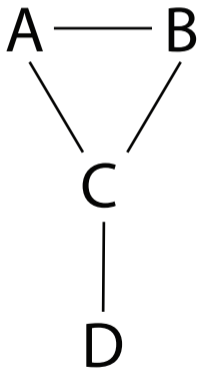


Directed Causal Structure

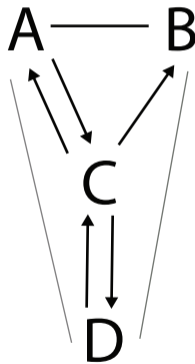


Project 1: Undirected Networks and Causal Skeletons

Undirected Network

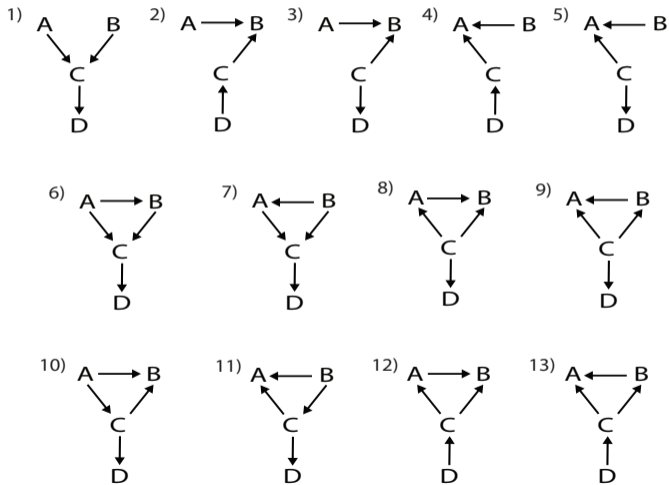
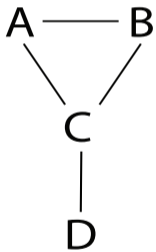


Directed Causal Structure

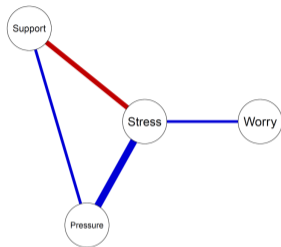


One-to-many mapping

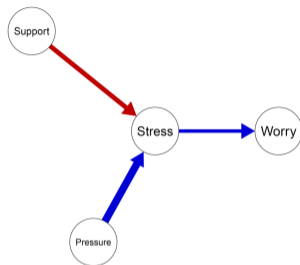
DAGs



SEset: Mapping GGMs to linear DAGs



$$\begin{array}{l}
 Su \quad P \quad St \quad W \\
 Su \quad \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \sigma_{14} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & \sigma_{24} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & \sigma_{34} \\ \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_{44} \end{bmatrix} \\
 P \\
 St \\
 W
 \end{array}$$



$$\hat{\Sigma}^{-1}$$



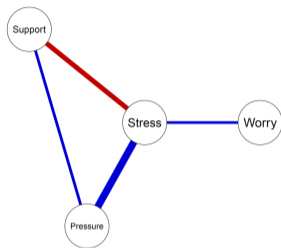
$$\hat{\Sigma}_{MI}$$



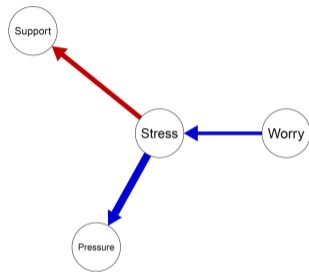
$$(I - B_j)^{-1} \Psi_j (I - B_j)^{-T}$$



SEset: Mapping GGMs to linear DAGs



$$\begin{array}{c}
 W \\
 St \\
 Su \\
 P
 \end{array}
 \begin{array}{cc}
 W & St & Su & P \\
 \left[\begin{array}{cccc}
 \sigma_{11} & \sigma_{12} & \sigma_{13} & \sigma_{14} \\
 \sigma_{21} & \sigma_{22} & \sigma_{23} & \sigma_{24} \\
 \sigma_{31} & \sigma_{32} & \sigma_{33} & \sigma_{34} \\
 \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_{44}
 \end{array} \right]
 \end{array}$$



$$\hat{\Sigma}^{-1}$$



$$\hat{\Sigma}_{MI}$$



$$(I - B_j)^{-1} \Psi_j (I - B_j)^{-T}$$



Example (Hoorelbeke et al. 2016)



(a) DAG #12



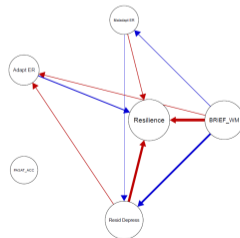
(b) DAG #52



(c) DAG #14



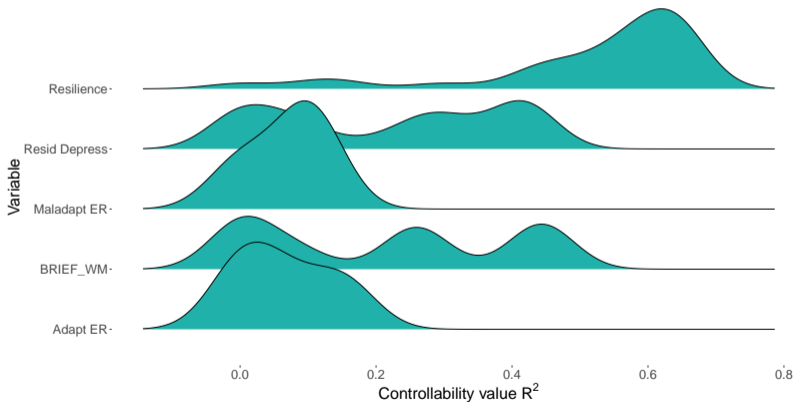
(d) DAG #20



Exploring Uncertainty

Predictability: R^2 if all variables cause Y

- ▶ “Upper bound” on controllability 2



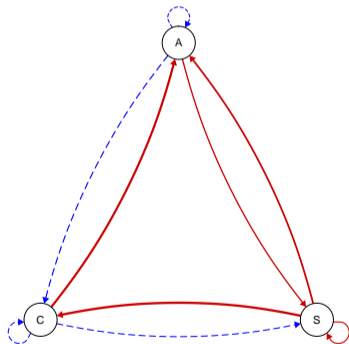
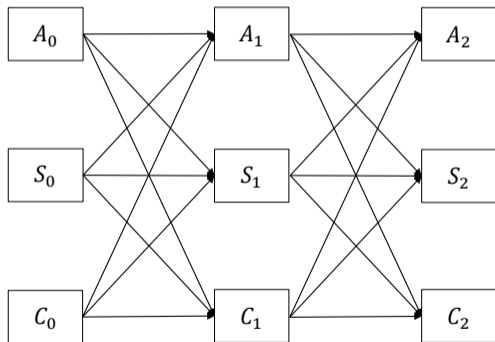
Future Directions 1

Beyond DAGs

- ▶ What other kinds of structures might cross-sectional networks be informative for?
- ▶ Dynamic structures - causal loops and systems in equilibrium
- ▶ Check undirected networks for consistency with theoretical models

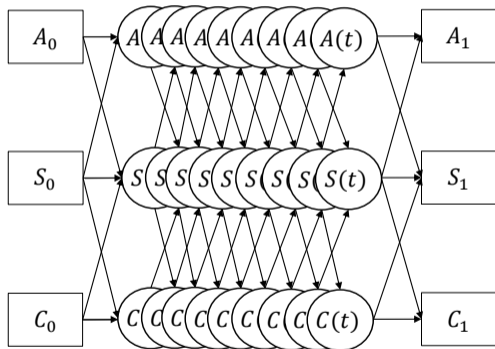
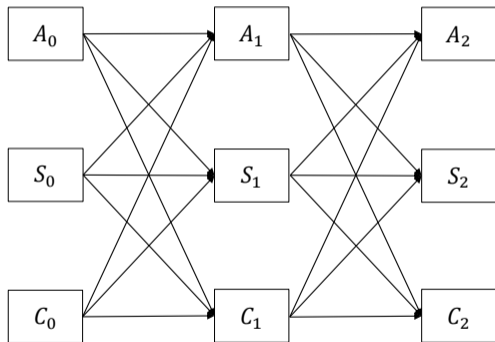
Project 2: Continuous-Time Modeling of ESM data

$$\mathbf{Y}_T = \mathbf{\Phi} \mathbf{Y}_{T-1} + \epsilon_T$$



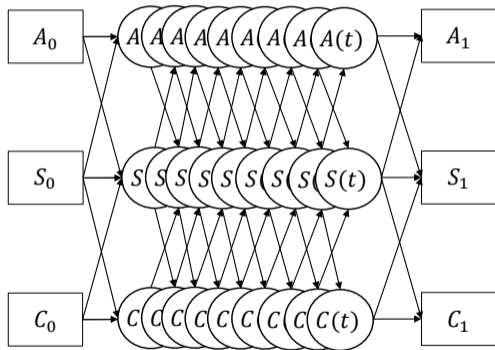
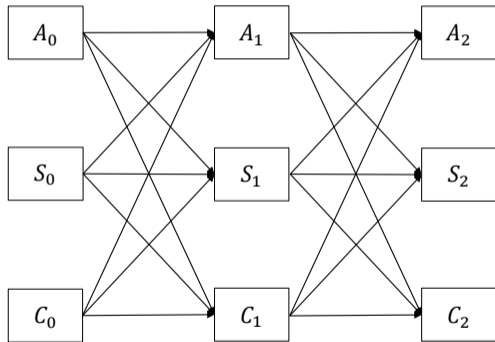
Project 2: Continuous-Time Modeling of ESM data

$$\mathbf{Y}_\tau = \Phi \mathbf{Y}_{\tau-1} + \epsilon_\tau$$



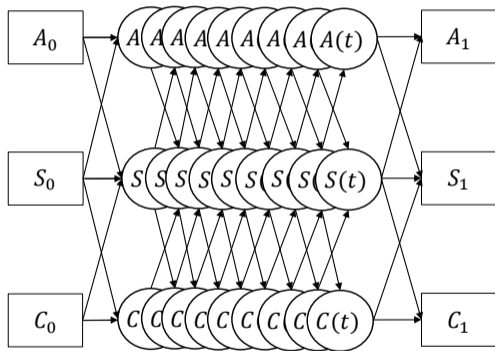
Project 2: Continuous-Time Modeling of ESM data

$$\frac{d\mathbf{Y}(t)}{dt} = \mathbf{A}\mathbf{Y}(t) + \epsilon$$



Implications of an underlying CT model

Implications of an underlying CT model



- ▶ Causal interpretation of VAR(1) paths misleading
- ▶ “Direct” effects made up of indirect pathways
- ▶ Centrality measures misinterpreted for any time-interval

Future Directions 2

Better selection of intervention targets

- ▶ Take into account time-interval dependency
- ▶ Place centrality measures, path-specific effects within a formal interventionist causal framework

Broaden the scope of CT models considered

- ▶ Build from theory - check with VAR(1) dependencies observed in real data
- ▶ One-to-many mapping to more complex dynamic models

Contact info

- ▶ `ryanoisin@gmail.com`
- ▶ `ryanoisin.github.io`