What paradox?

Causal Models and Statistical Confusion

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Male		
Female		

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Statistical phenomena where a relationship which is present when aggregating over the population may be reversed or absent when looking at sub-populations

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The relationship between a categorical exposure and a continuous outcome is reversed when we condition on a third variable

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Confusing, but **not a paradox**

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You're asking a question that statistics alone is not equipped to answer

Estimand	Estimator	Estimate

Estimand **Estimator** **Estimate**

Statistics in a nutshell

Estimand

Estimator

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Remove your cooled cakes from their pans and level them with a ruler and serrated knife.

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Credit to Peter Tennant @PWGTennant

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Conditional Probabilities:

$$P(R = r | D = d, S = s)$$

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Marginal Probabilities:

$$P(R = r|D = d)$$

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Statistics in a nutshell

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$P(R = 1 \mid D = 1)$	# Recovered drug takers / # Drug takers	$.78$

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Two different sets of **estimands** yield two different sets of **estimates**

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- Does taking the drug cause recovery?
- **Causal Estimand**
- But we have no way of expressing this in the language of statistics

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Statistical estimand $\leftarrow$? $\rightarrow$ Causal Estimand

Causal Graphs

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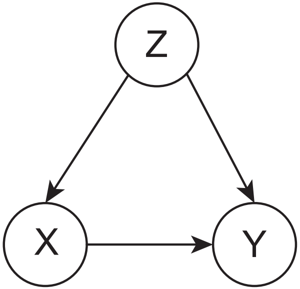
- The arrow $X \to Y$ represents our belief that X is a direct cause of Y
- We omit an arrow if expert knowledge tells us that one variable does not directly cause another. The *absence* of an arrow is a strong statement



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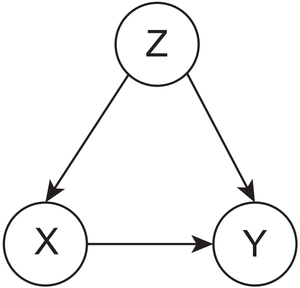

Directed Acyclic Graph (DAG) or Bayesian Network

Why Causal Models?

This machinery is useful for three important and closely related reasons:

1. Causal models map causal dependencies onto statistical dependencies
   - *Regardless* of distributions and functional forms
2. Causal models allow us to define **causal effects** in the language of interventions and probabilities
3. Causal models tell us which when and how statistical estimands can act as causal estimands

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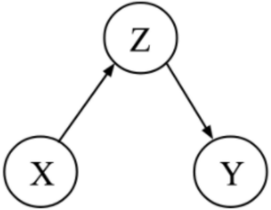
Chain

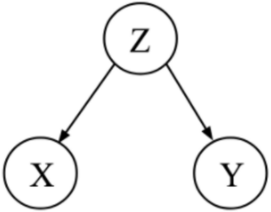


X: Smoking
Z: Tar
Y: Cancer

$X \not\perp\!\!\!\perp Y$

$X \perp\!\!\!\perp Y \mid Z$

Fork



X: Storks
Z: Environment
Y: Babies

$X \not\perp\!\!\!\perp Y$

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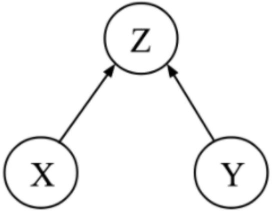
Collider



X: Attractiveness
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Y: Intelligence

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Causal Effects in SCMs

The **do-operator** $do(X = x)$ represents a "surgical intervention" to set the value of the variable X to a constant value x

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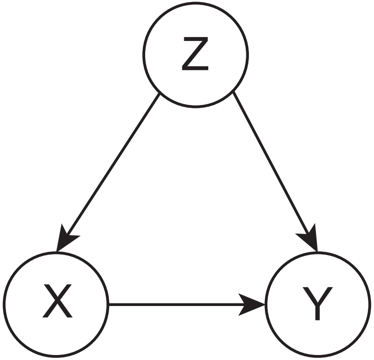
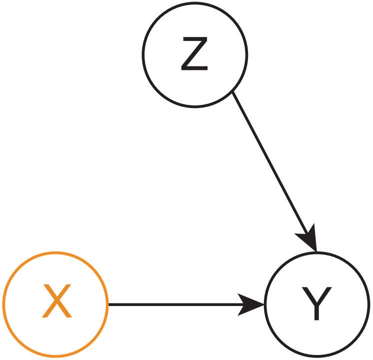
In the graph, a $do-$ operation on X cuts-off all incoming ties

Two versions of the causal system

Observing



Intervening



Causal Effects in SCMs

We can use the do-operator to define our **causal estimand**

Causal Effect of Drug-Taking on Recovery:

$$CE = P[R \mid do(D = 1)] - P[R \mid do(D = 0)]$$

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Inference problem: “Seeing” is not always the same as ”doing”

Observing $\neq$ Intervening:

$P[Y \mid X = x]$ is not **generally** the same as $P[Y \mid do(X = x)]$

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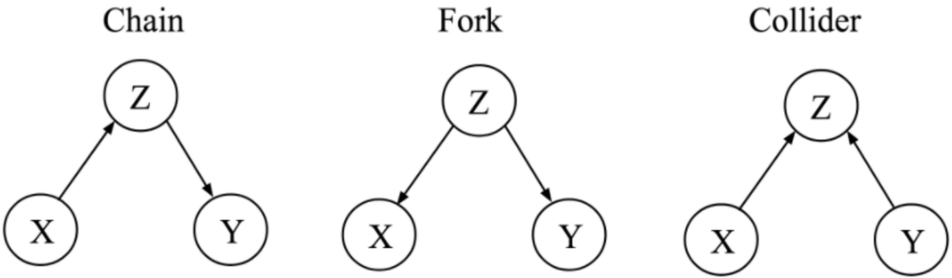

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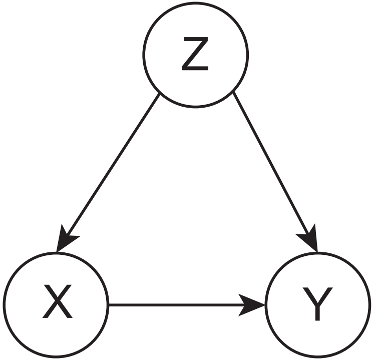

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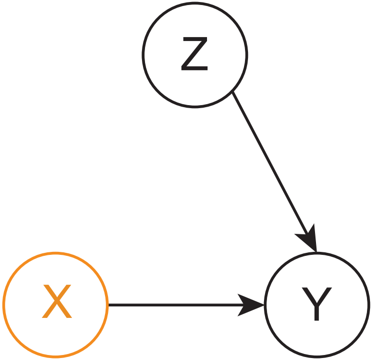

Statistics in a nutshell

Statistical Estimand

Estimator

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Causal Estimand	**Causal Model**	**Statistical Estimand**	**Estimator**	**Estimate**

Causal Inference in a nutshell

Causal Estimand

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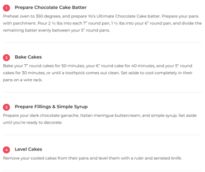

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Causal Inference in a nutshell

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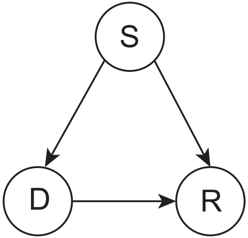
$$P[R \mid do(D=1)] - P[R \mid do(D=0)]$$

Causal Model



Statistical Estimand

$$P(R|D,S)$$

$$P(R|D)$$

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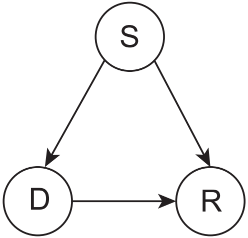


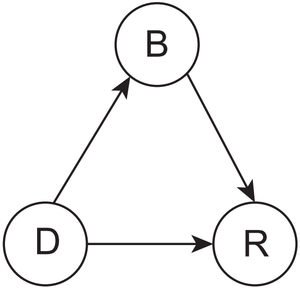
Statistical Estimand

$$P(R|D,S)$$

$$\cancel{P(R|D)}$$

Post-Treatment Blood Pressure:

- Statistical information is exactly the same
- The drug works in part by decreasing blood pressure
- We should **not** condition on blood pressure



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Example from Pearl, Glymour & Jewell, 2016

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Absolutely not a paradox.

- Confusion comes from a lack of clarity regarding our **causal estimand** and **causal model**

Statistical information *alone* cannot provide the answer

- Different DAGs can produce the exact same statistical dependencies in observational data

Causal models provide immediate conceptual clarity

- Miguel Hernan: Draw your assumptions before your conclusions!

Conclusions

Inappropriate reliance on (advanced) statistical modeling with no clear link to causal estimands or models

- Paradoxes and confusion result. Machine learning is no solution

Causal modeling can be powerful in reshaping how we approach statistical modeling

- Judea Pearl, Don Rubin, Jamie Robins, Miguel Hernan, Angrist & Imbens
- Example: Controlling for as many variables as possible is **an obviously terrible idea** when estimating causal effects

Researchers make causal inferences based on observational data **all the time**

- Better to be explicit and open about this so we can move forward

Thanks!
(o.ryan@uu.nl | oisinryan.org)

Shameless plug

My own research focuses on using these ideas to improve psychological and social science research

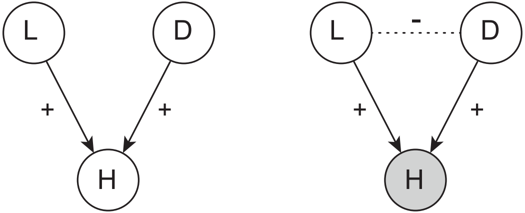
- Causal discovery (e.g. Ryan, Bringmann, Schuurman, in press)
- Causal estimands (e.g. Haslbeck*, Ryan*, Dablander* 2021)
- Constructing theories (Haslbeck*, Ryan*, Robinaugh*, Waldorp, Borsboom, 2021)
- Applications of causal inference (forthcoming)

Berksons Paradox

Two phenomena which are statistically *independent* in the general population are statistically *dependent* in a sub-population

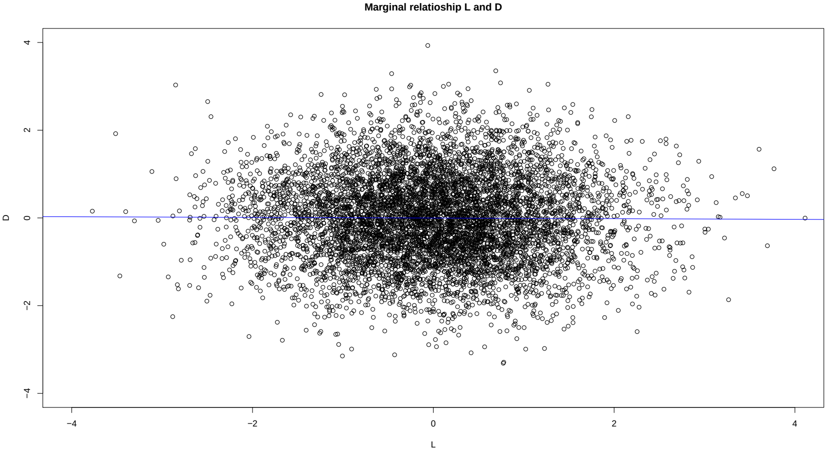
Classic example: We are interested in the relationship between *Lung Cancer* (L) and *Diabetes* (D)

- General population, these two variables are independent.
- In a sample of *hospital patients*, there is a negative dependency - patients who don't have diabetes are *more likely* to have lung cancer.



- Lung cancer L and diabetes D cause hospitalization H
- By taking participants from a hospital we *condition* on hospitalization $(H = 1)$
- If you are hospitalised, and you *don't* have diabetes, probably you do have lung cancer (Otherwise - why would you be in hospital?).
- $P(D|L = 1, H = 1) \neq P(D|do(L) = 1)$
- We have conditioned on a *collider*

Collider Bias



Collider Bias