Dynamical network analysis: A Continuous-Time Approach

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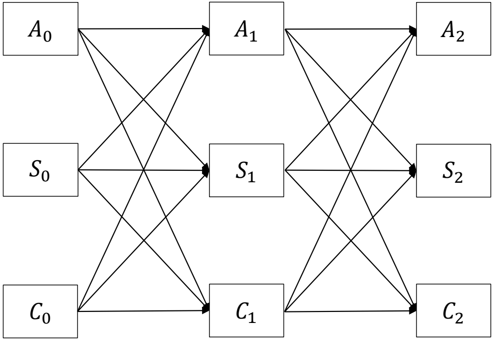
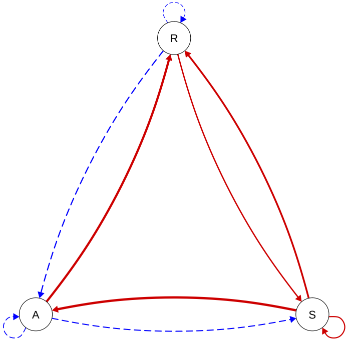
- The *Continuous-Time* VAR(1) model is a very appealing alternative
  - Overcomes the problem of time-interval dependency
  - Lets us model **how** lagged effects change with the time-interval
  - Matches closer with our substantive ideas about what psychological processes are

Dynamical networks and the VAR(1) model

$$\boldsymbol{Y}_{\tau} = \boldsymbol{\Phi} \boldsymbol{Y}_{\tau-1} + \boldsymbol{\epsilon}_{\tau}$$





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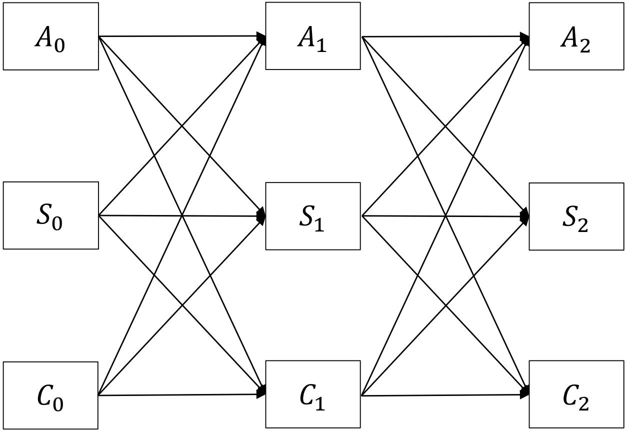

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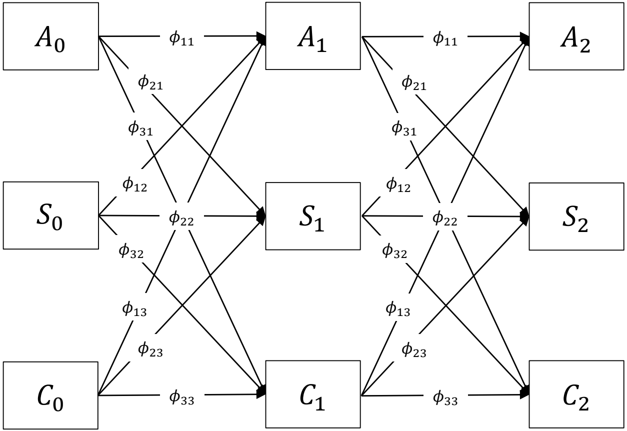

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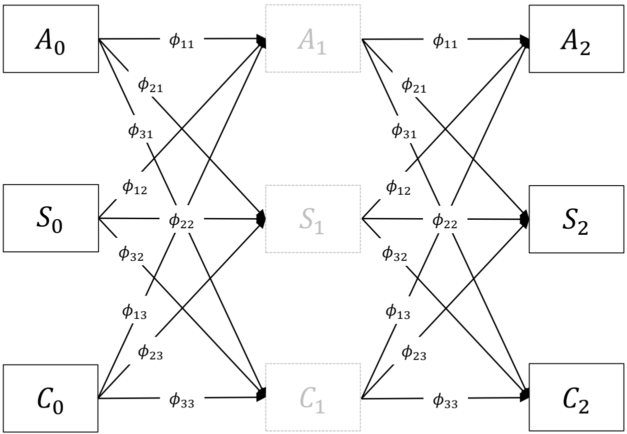

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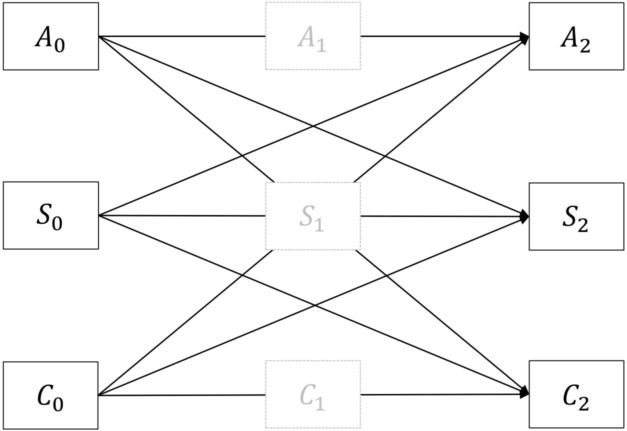

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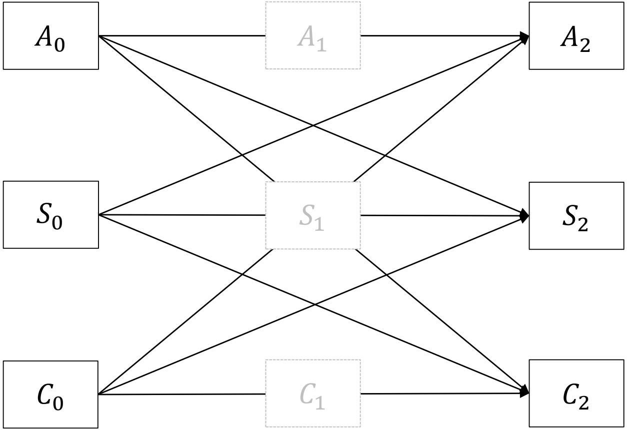

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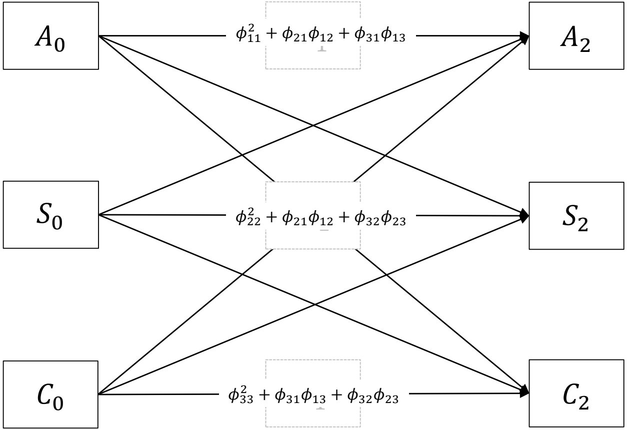
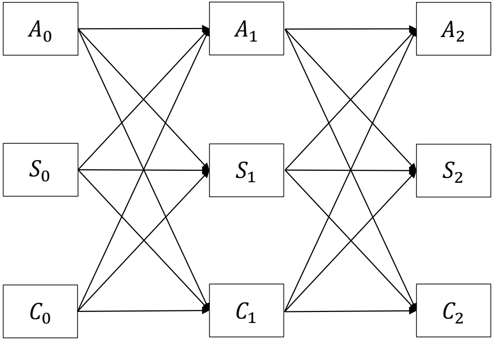
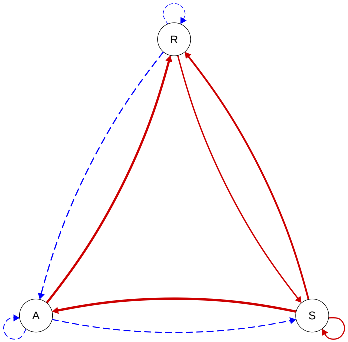


Implications for Dynamical Network Structure





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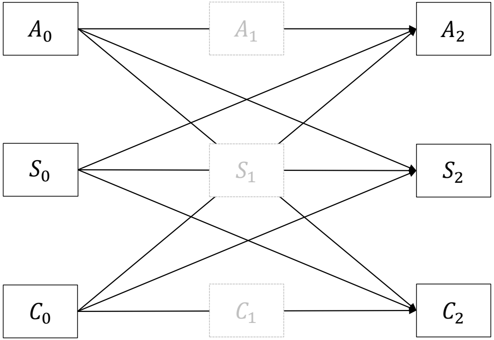

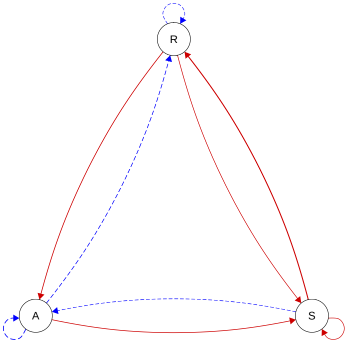

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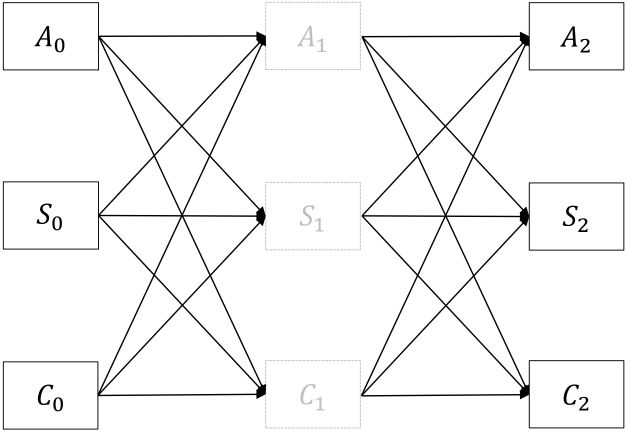

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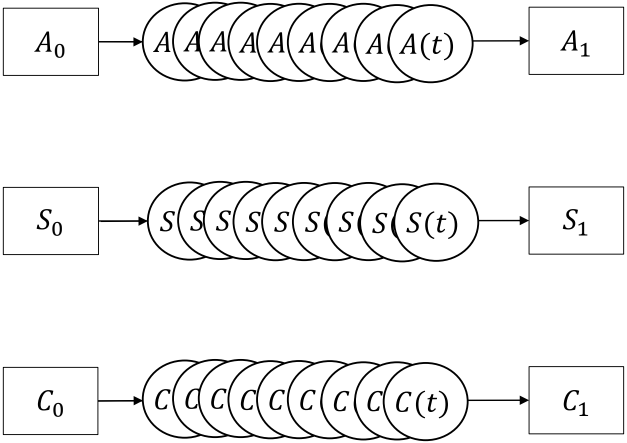

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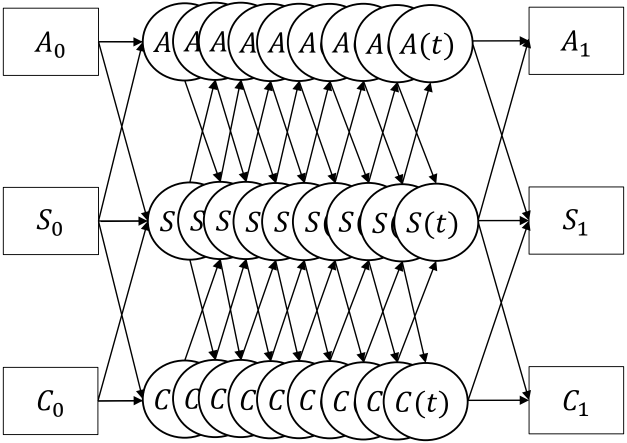

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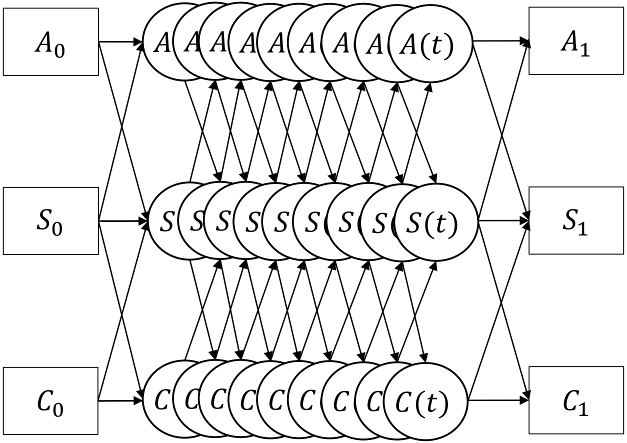

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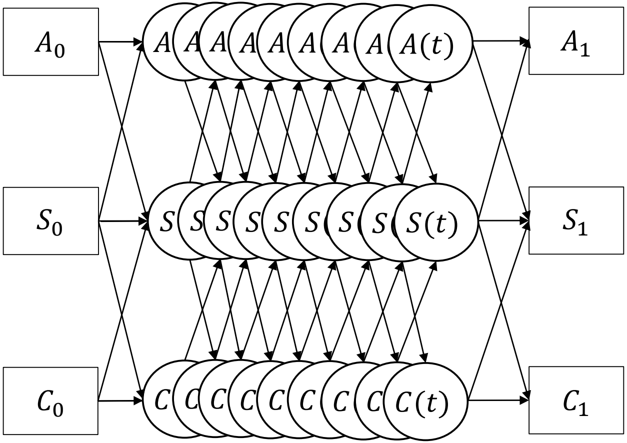

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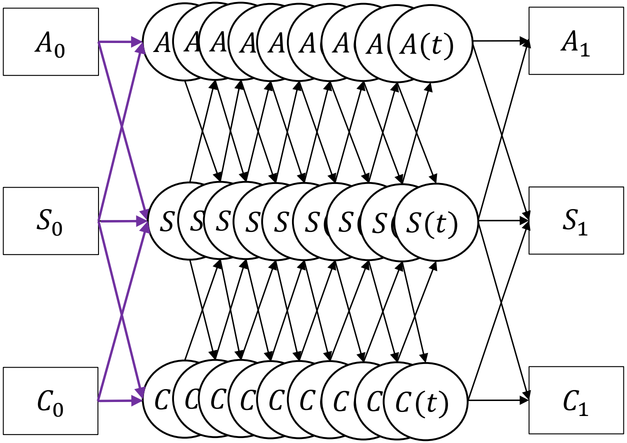

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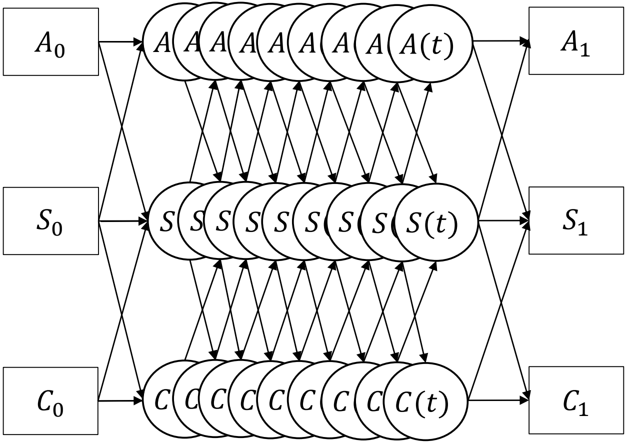

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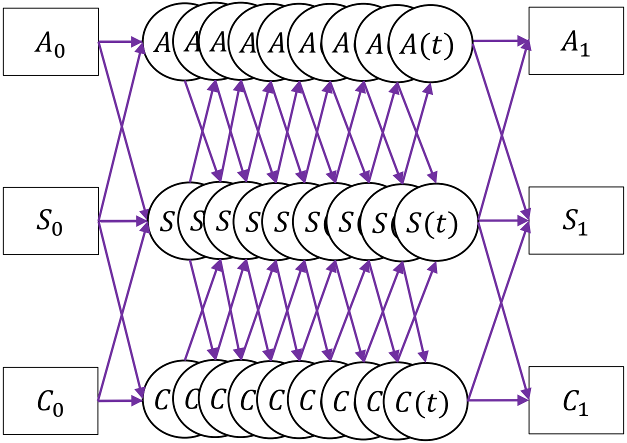
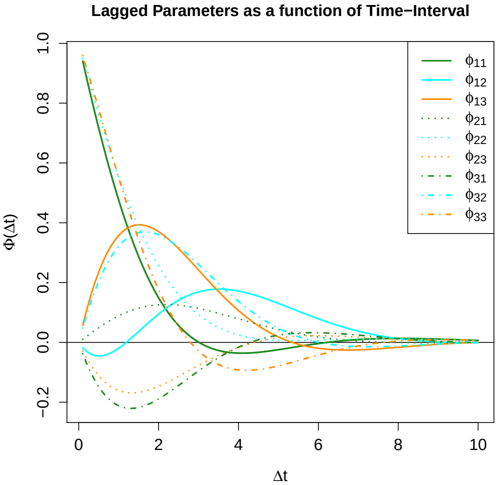


Time-interval dependency of VAR estimates

$$e^{\boldsymbol{A}\Delta t} = \boldsymbol{\Phi}(\Delta t)$$



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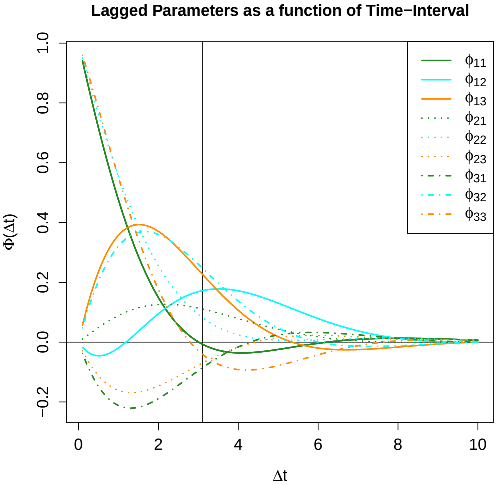

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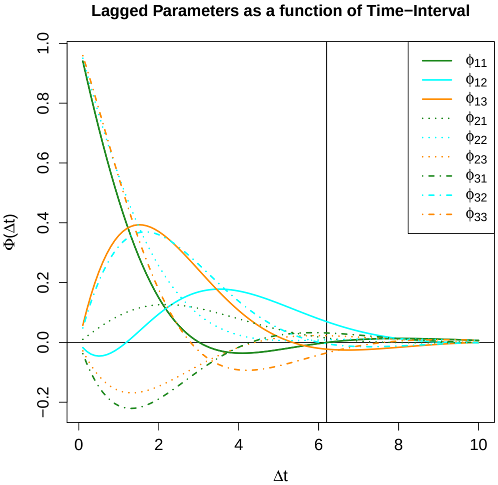
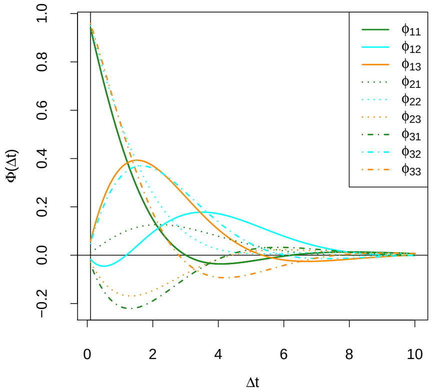
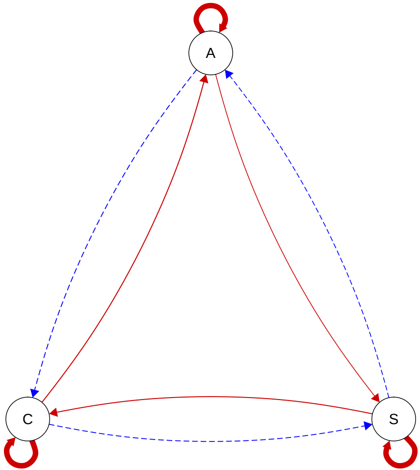


Network structure as a function of lag





From the Why to the How: Estimation

- CT VAR(1) model is based on a first-order differential equation
- Boker, Oud, Voelkle and many others have argued for these models in psychology
- Many exciting estimation possibilities
  - ctsem - Driver, Voekle, Oud
  - GLLA and LDE through OpenMx - Boker and colleagues
  - BHOU - Oravecz and colleagues
  - Indirect estimation (using DSEM in Mplus)*
  - Extended Multi-level CT models - Rebecca Kuiper *

Further Implications/Discussion

- Problem is not just with VAR(1) parameter **estimates** but also their interpretation
  - $\boldsymbol{\Phi}$ parameters are not **direct** links in the intuitive sense
- How do we find "the" network structure
  - Drift matrix directly vs summary measures based on $\boldsymbol{e}^{\boldsymbol{A}\Delta t}$
- Centrality measures
  - Adapt existing or make new ones
  - Clarify their interpretation / substantive importance

Get in Touch

- http://dml.sites.uu.nl/
- o.ryan@uu.nl

Continuous Time Model

First-Order Stochastic Differential Equation

$$\frac{d\boldsymbol{Y}(t)}{dt} = \boldsymbol{A}(\boldsymbol{Y}(t) - \boldsymbol{\mu}) + \gamma \frac{d\boldsymbol{W}(t)}{dt}$$

CT VAR(1) Model

$$\boldsymbol{Y}(t) = \boldsymbol{e}^{\boldsymbol{A}\Delta t}\boldsymbol{Y}(t - \Delta t) + \boldsymbol{w}(\Delta t)$$

Numerical Example

$$\boldsymbol{A} = \begin{bmatrix} -6 & -.2 & .6 \\ .1 & -.5 & -.3 \\ -.4 & .5 & -.4 \end{bmatrix}$$

Application to Empirical Data

- N=1 Experience Sampling Data
- Geschwind et al. (2011)
- 115 repeated measurements
  - *Perceived Unpleasantness* (PU)
  - *Worry* (W)
  - *Relaxation* (Re)

$$\boldsymbol{A} = \begin{bmatrix} -2.423 & 0.177 & -0.200 \\ 1.140 & -2.445 & -1.964 \\ -0.616 & 0.204 & -0.884 \end{bmatrix}$$

Results Empirical Data