Mediation and Causal Mechanisms: A Continuous-Time Approach

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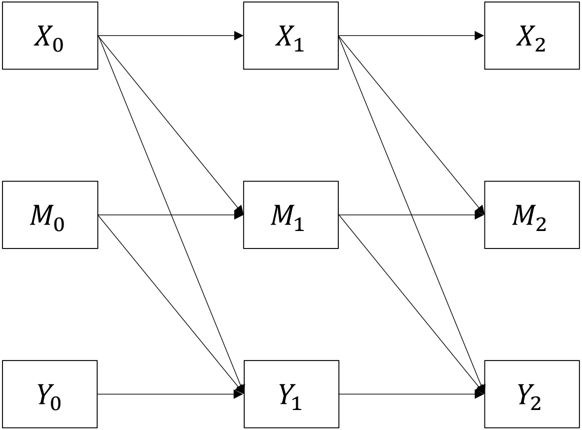
Longitudinal Mediation

Cole & Maxwell (2003, 2007)



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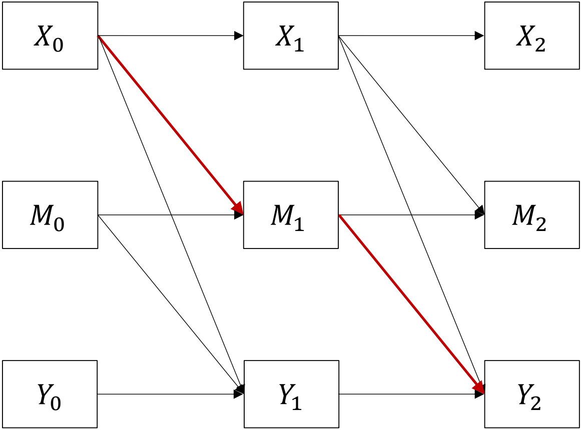

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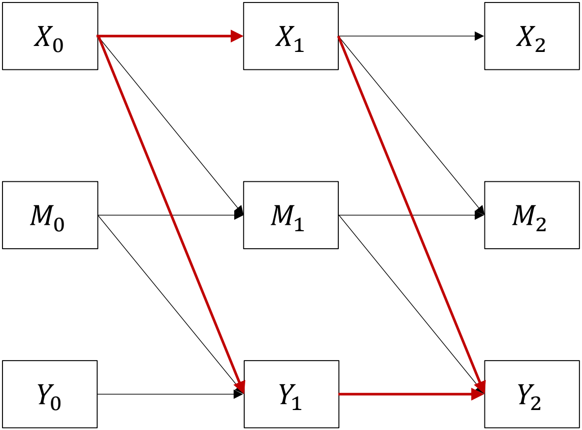

The Discrete-Time VAR(1) model

$$\boldsymbol{Z}_\tau = \boldsymbol{\Phi} \boldsymbol{Z}_{\tau-1} + \boldsymbol{\epsilon}_\tau$$

- Cross-Lagged Panel Model (**CLPM**; Cole & Maxwell, 2003) or First-Order Vector Auto-regressive (**VAR(1)**; Hamilton (1994)) model
- Characterized as a **Discrete-Time** model; time is accounted w.r.t the order of measurement only

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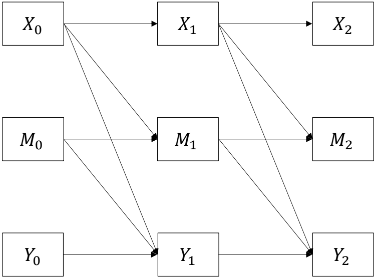
Causal Interpretation of path-specific effects

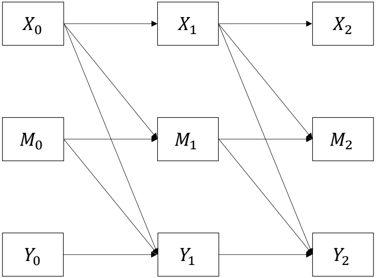
Interventionist Causal Framework (Pearl, Rubin, Robins amongst others)

- Causal effects $\rightarrow$ **interventions on variables** in our model
  - Effects of (possibly hypothetical) experiments
- *If certain assumptions hold*, we can identify the effect of an intervention without *necessarily* performing that experiment
- Once we can make these assumptions explicit we can explore whether they are realistic or not

Controlled Direct Effect (VanDerWeele 2015)

$$\mathbf{CDE} = E(Y_2|X_0 = 1, M_1 = 0) \quad - \quad E(Y_2|X_0 = 0, M_1 = 0)$$





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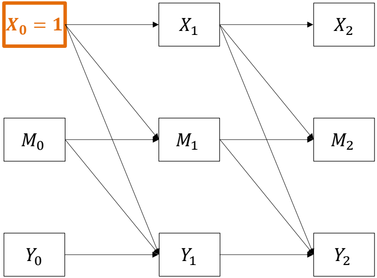

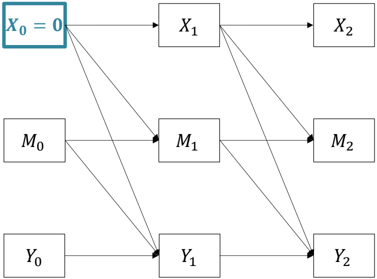

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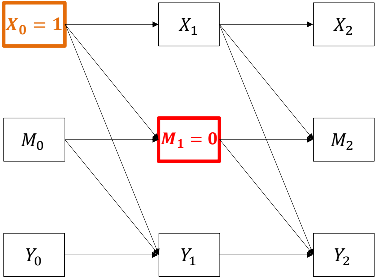

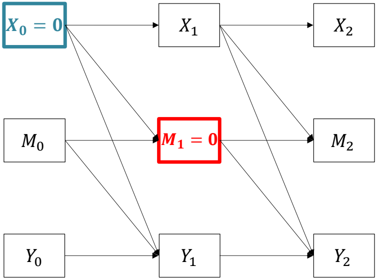

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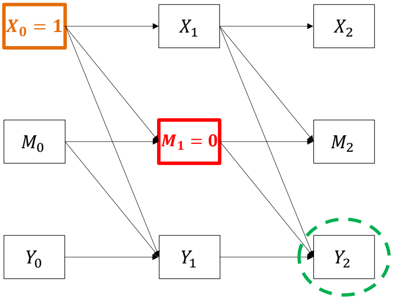

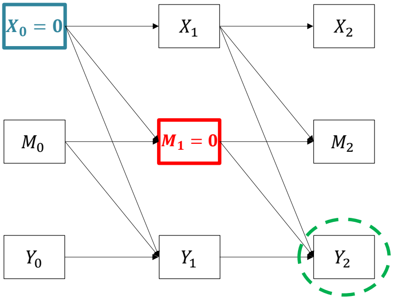

Assumptions needed

- No unobserved common cause of X and Y at any occassion(s) τ
- No unobserved common cause of M and Y at any occassions(s) τ

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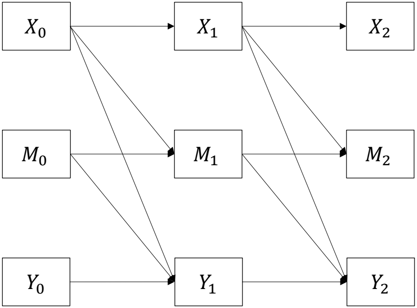
Using $\mathbf{\Phi}$ the CDE equals path-tracing direct effect of X_0 on Y_2

An alternative dynamical model



- DT models unrealistic
- Psychological variables (e.g. Stress, Affect) do not evolve in discrete steps
- Vary in a **continuous** manner over time (Boker 2001)

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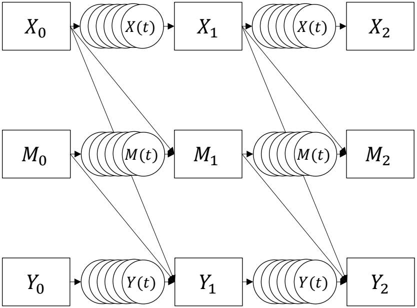

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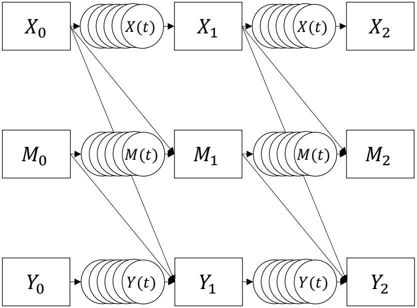

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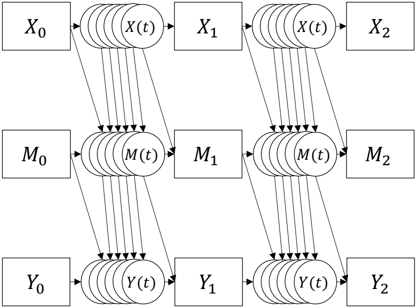

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An alternative dynamical model

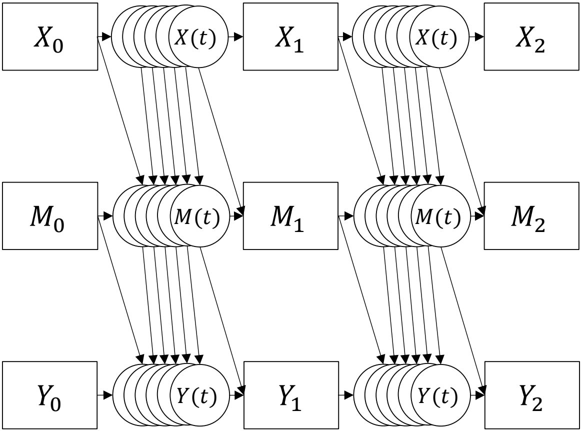
Continuous-Time VAR(1) Model

Based on a first-order differential equation (Boker et al., 2010, Voelkle, Oud et al., 2012, Oravecz et al., 2009).

$$\boldsymbol{e}^{\boldsymbol{A}\Delta t} = \boldsymbol{\Phi} \tag{1}$$

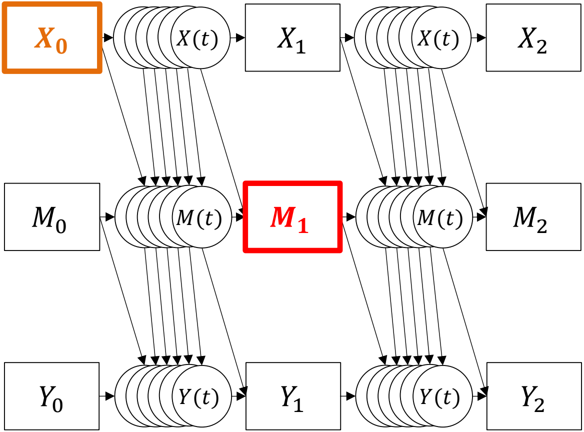
Deboeck & Preacher (2016) - CT-VAR(1) for mediation

Interventions and CT



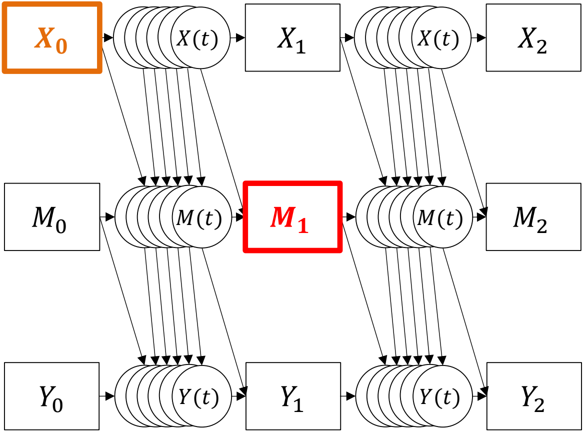
Interventions and CT processes

$$CDE = E(Y_2|X_0 = 1, M_1 = 0) - E(Y_2|X_0 = 0, M_1 = 0)$$



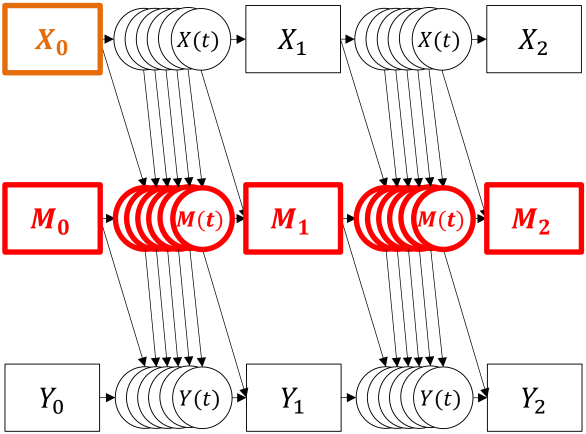
Interventions and CT processes

Acute Intervention



Interventions and CT processes

Interval Intervention



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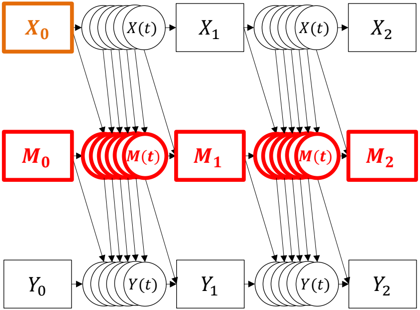

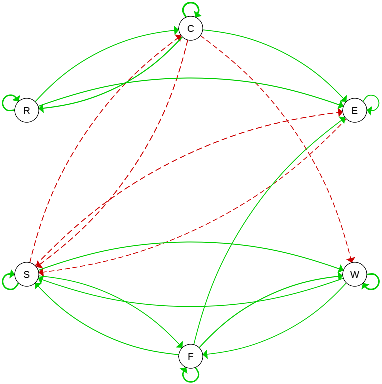

Summary

- Mediation is a fundamentally causal concept
- The interventionist framework helps us to make explicit what path-specific effects mean
- CT models help in specifying and exploring different types of interventions

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Proof equivalence of path and variable setting

Deboeck & Preacher suggest finding direct effects by disabling paths in the $v \times v$ drift matrix $\boldsymbol{A}$ before applying the matrix exponential term.

Take $\boldsymbol{S}$ to be an intervention matrix; equivalent to an identity matrix with one diagonal element set to zero. E.g., if we are interested in an intervention on M, let $\boldsymbol{S} = diag(1, 0, 1)$.
$\boldsymbol{S}$ is nilpotent, thus $\boldsymbol{S}.\boldsymbol{S} = \boldsymbol{S}$

Setting the initial end final values of M in the interval to zero, this definition of the direct effect is

$$\boldsymbol{S}.e^{\boldsymbol{S}.\boldsymbol{A}.\boldsymbol{S}\Delta t}.\boldsymbol{S}$$

Proof 1

It suffices to show that

$$\boldsymbol{S}.e^{\boldsymbol{S}.\boldsymbol{A}.\boldsymbol{S}\Delta t}.\boldsymbol{S} = \lim_{k\to\infty}(\boldsymbol{S}.e^{\boldsymbol{A}\Delta t/k}.\boldsymbol{S})^k \tag{2}$$

$$\boldsymbol{S}.e^{\boldsymbol{S}.\boldsymbol{A}.\boldsymbol{S}\Delta t}.\boldsymbol{S}$$

$$\boldsymbol{S}.\boldsymbol{I}.\boldsymbol{S} + \boldsymbol{S}.\boldsymbol{A}.\boldsymbol{S}\Delta t + \frac{\boldsymbol{S}.\boldsymbol{A}.\boldsymbol{S}^2(\Delta t)^2}{2!} + \dots$$

$$\lim_{n\to\infty}(\boldsymbol{S}.\boldsymbol{I}.\boldsymbol{S} + \frac{\boldsymbol{S}.\boldsymbol{A}.\boldsymbol{S}\Delta t}{n})^n$$

$$\lim_{k\to\infty}(\boldsymbol{S}.e^{\boldsymbol{A}\Delta t/k}.\boldsymbol{S})^k$$

As $k \to \infty$

$$e^{\boldsymbol{A}\Delta t/k} \to \boldsymbol{I} + \frac{\boldsymbol{A}\Delta t}{k}$$

$$\lim_{k\to\infty}(\boldsymbol{S}(\boldsymbol{I} + \frac{\boldsymbol{A}\Delta t}{k})\boldsymbol{S})^k$$

$$\lim_{n\to\infty}(\boldsymbol{S}.\boldsymbol{I}.\boldsymbol{S} + \frac{\boldsymbol{S}.\boldsymbol{A}.\boldsymbol{S}\Delta t}{n})^n$$

Continuous Time Model

First-Order Stochastic Differential Equation

$$\frac{d\boldsymbol{Z}(t)}{dt} = \boldsymbol{A}(\boldsymbol{Z}(t) - \boldsymbol{\mu}) + \gamma \frac{d\boldsymbol{W}(t)}{dt}$$

CT VAR(1) Model

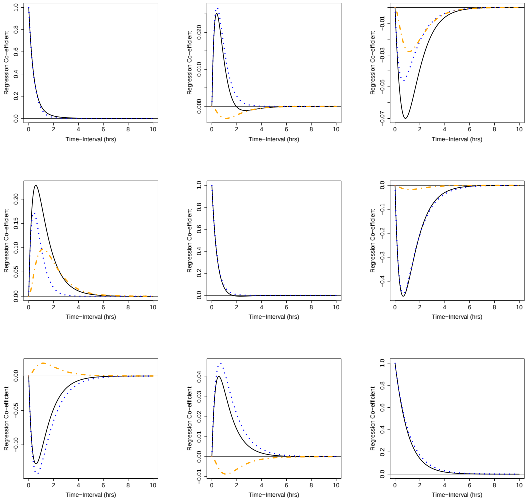
$$\boldsymbol{Z}(t) = \boldsymbol{e}^{\boldsymbol{A}\Delta t} \boldsymbol{Z}(t - \Delta t) + \boldsymbol{w}(\Delta t)$$

Application to Empirical Data

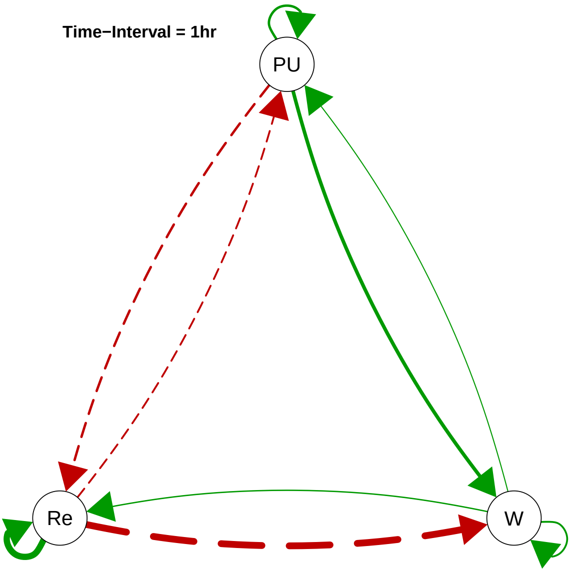
- N=1 Experience Sampling Data
- Geschwind et al. (2011)
- 115 repeated measurements
  - *Perceived Unpleasantness* (PU)
  - *Worry* (W)
  - *Relaxation* (Re)

$$\boldsymbol{A} = \begin{bmatrix} -2.423 & 0.177 & -0.200 \\ 1.140 & -2.445 & -1.964 \\ -0.616 & 0.204 & -0.884 \end{bmatrix}$$

Results Empirical Data



Results - DT Network



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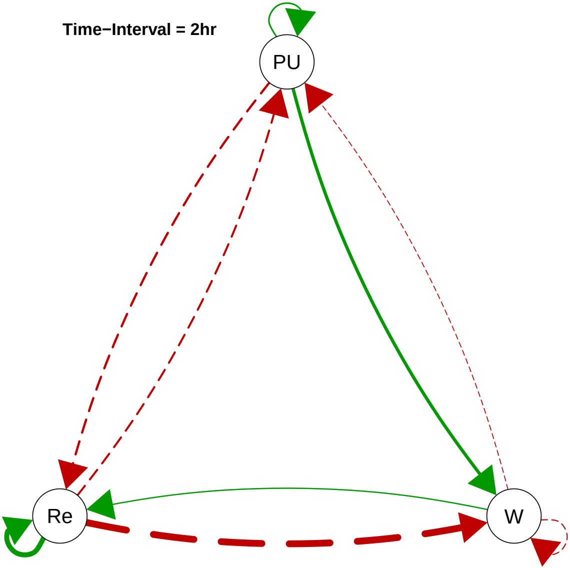
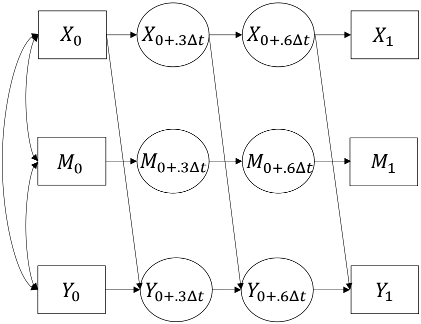


Indirect, Direct and Total Effects

Continuous Time Framework



$$\mathbf{\Phi}(\Delta t_\tau) = \begin{bmatrix} \phi_{11} & 0 & 0 \\ 0 & \phi_{22} & 0 \\ DE & 0 & \phi_{33} \end{bmatrix}$$

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ a_{31} & 0 & a_{33} \end{bmatrix}$$

Indirect, Direct and Total Effects

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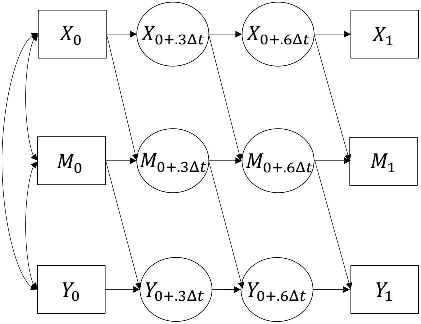

$$\boldsymbol{\Phi}(\Delta t_\tau) = \begin{bmatrix} \phi_{11} & 0 & 0 \\ \phi_{21} & \phi_{22} & 0 \\ IE & \phi_{32} & \phi_{33} \end{bmatrix}$$

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